OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
Nov. 15, 1996

3. REPORT TYPE AND DATES COVERED
Final 1 Aug 94 - 31 Aug 96

4. TITLE AND SUBTITLE
Level sets and stochastic partial differential equations

5. FUNDING NUMBERS
DAAH04-94-G-0261

6. AUTHOR(S)
Robert C. Dalang

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES)
Tufts University, Medford, MA 02155

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
ARO 32888.7-MA

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The effort reported on here was primarily aimed at acquiring a better understanding of a broad class of stochastic partial differential equations.

The main class of problems was concerned with regularity properties of solutions to stochastic wave equations in one and two spatial dimensions. A second class of problems arose from attempts to understand the flow of information thoughout the solution of a linear stochastic wave equation in two spatial dimensions driven by Lévy (shock) noise. A third topic studied was in the area of stochastic optimization.

Substantial results have been obtained in all three areas. These results have given rise to six published (or soon to be published) research articles, a published monograph and a Ph.D. thesis.

14. SUBJECT TERMS
Level sets, Brownian sheet, wave equation, stochastic partial differential equations

15. NUMBER IF PAGES
6

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18
298-102

LEVEL SETS

AND

STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

FINAL PROGRESS REPORT

Robert C. Dalang

August 31, 1996

U.S. ARMY RESEARCH OFFICE

Grant No. DAAH-04-94-G-0261

Tufts University

A. Statement of the problem studied

The effort reported on here was primarily aimed at acquiring a better understanding of a broad class of stochastic partial differential equations. These equations are an important modelling tool for describing the evolution of physical media subject to random excitation.

The main class of problems was concerned with regularity properties of solutions to stochastic wave equations. In one spatial dimension, deep sample path properties, such as existence of points of increase and non-differentiability of solutions along curves were considered. In two spatial dimensions, continuity and short-term existence of solutions to a non-linear stochastic wave equation driven by non-white Gaussian noise were examined. The emphasis on non-white noise is a departure from most of the existing literature on such equations. This emphasis is motivated by the fact that in many applications [3, 18], Gaussian noise with non-degenerate spatial correlation is a better model than white noise.

A second class of problems arose from attempts to understand the flow of information thoughout the solution of a linear stochastic wave equation in two spatial dimensions driven by Lévy (shock) noise. Most of these questions had been previously examined by the Principal Investigator in the context of one dimensional space, but new methods were required to move up to two dimensional space.

A third direction of research was concerned with certain stochastic optimization problems. The goal of this type of research is to understand how to best make use of available information in a stochastic decision making environment.

B. Summary of the most important results

THE ONE-DIMENSIONAL WAVE EQUATION. In article 2 (see the List of all publications and technical reports), the PI and T. Mountford (UCLA) have shown that the sample paths of the Brownian sheet belong with probability one to the space of functions of two variables that admit strict points of increase along monotone curves. This shows that the analogue for the Brownian sheet of the famous 1961 result of Dvoretzky, Erdös and Kakutani for Brownian motion does not extend to the Brownian sheet, and therefore that the structure of excursions of the Brownian sheet is quite different from what was previously thought.

In article 4, it is shown that the function space just mentioned has Baire category one and is therefore a "small" set, whereas the result obtained above indicates that it is a "large" set. This difference does not occur with Brownian motion, for which most sets of positive measure are of second Baire category. This Baire category result extends a result of Bruckner and Garg [2] concerning functions of one real variable.

In article 3, the PI and T. Mountford have shown that the Brownian sheet is nowhere differentiable in any direction in the strongest possible sense: given any Jor-

dan arc, there are no points at which the arc has a tangent and the sheet viewed along the arc is differentiable at that specific point. This result implies all previously known results on this problem [6], and also has important implications for our understanding of geometric properties of level sets of the Brownian sheet.

THE TWO DIMENSIONAL WAVE EQUATION. In article 5, the PI and N. Frangos have completed their study of the stochastic wave equation in two spatial dimensions driven by non-white Gaussian noise. A necessary and sufficient condition on the covariance function of the noise process for existence of a jointly measurable mean-square bounded solution has been obtained. Examples of processes that satisfy this condition are provided, including the so-called "Brownian free field." Since the last progress report, the situation with regard to non-linear forms of this equation has been clarified. The condition that is necessary and sufficient in the case of the linear equation also suffices to guarantee short-term existence of solutions to non-linear forms of the equation. The question of long-term existence remains open (except in the case of the linear equation).

In article 6, the PI and and his former Ph.D. student Q. Hou study the sharp Markov property for solutions of the two dimensional wave equation driven by a locally finite Lévy point process. The solution to this equation is the superposition of waves created by distinct and well localized shocks.

In this article, they establish this Markov property for domains that are bounded polyhedra. Previously, they had established this property for domains bounded by half-spaces, that correspond to the line of sight of an observer moving at a speed greater than the speed of waves. Even though the sample paths of the solution process are discontinuous, this result is proved using properties of real-analytic functions of two variables. In addition, it is shown that the germ-field of the boundary of an open set actually contains all information about the process in the past light-cone of the domain. These results are part of Q. Hou's Ph.D. thesis, titled "On Markov properties of Solutions to Wave Equations Driven by Lévy Noise." Q. Hou's degree was awarded by Tufts University in May 1996. The question of whether or not the sharp field of the boundary of an arbitrary bounded open set also has this property remains open.

STOCHASTIC OPTIMIZATION. In the monograph 7, a unified mathematical theory of sequential stochastic optimization is presented. The monograph emphasizes problems of optimal stopping and control of stochastic processes in the presence of incomplete information, together with several applications, including sequential statistical testing involving several populations and the multi-armed bandit problem. Much of the material presented here is either new or appears in a book for the first time. Most of the main results were obtained over a period of several years, many of which were described in the Final report of the grant DAAL-03-92-6-0323. Dur-

ing the period of this grant, the remaining work on this monograph concerned the time-consuming preparation and actual publication of the final version of the book. Several stochastic control problems that involve optimal switching between two or more observation processes and for which results obtained in this monograph are useful are under continuing investigation.

C. List of all publications and technical reports

Research articles

1. R. Cairoli and R.C. Dalang, Optimal Switching Between Two Random Walks, *Annals of Probability,* 23 (1995), 1982-2013.
2. R.C. Dalang and T. Mountford, Points of Increase of the Brownian Sheet, *Probab. Th. and Related Fields* (28 pages, to appear).
3. R.C. Dalang and T. Mountford, Non-differentiability of curves on the Brownian sheet, *Annals of Probability,* 24-1 (1996), 182-195.
4. R.C. Dalang and T. Mountford, Points of increase of functions in the plane, *Real Analysis Exchange* (9 pages, to appear).
5. R.C. Dalang and N. Frangos, The stochastic wave equation in two spatial dimensions (25 pages, submitted).
6. R.C. Dalang and Q. Hou, On Markov properties of Lévy waves in two dimensions (23 pages, submitted).

Monograph

7. R. Cairoli and R.C. Dalang, *Sequential Stochastic Optimization,* (10 chapters, xii + 327 pages, 1996), Wiley, New York.

D. List of all scientific personnel showing any advanced degrees earned by them while employed on the project

R. Dalang (PI)
Q. Hou, Ph.D, May 1996

REPORT OF INVENTIONS (BY TITLE ONLY)

None.

References

[1] Alabert, A., Ferrante, M. & Nualart, D., Markov field property for stochastic differential equations. *Annals of Probab.* 23, 1995, 1262-1288.

[2] Bruckner, A.M. & Garg, K. (1977). The level set structure of a residual set of continuous functions. *Trans. Amer. Math. Soc.* **232**, 307-321.

[3] Biswas, S.K. & Ahmed, N.U., Stabilisation of systems governed by the wave equation in the presence of distributed white noise, *IEEE Trans. on Aut. Control,* AC-30 (1985).

[4] Carmona R. & Nualart, D., Random nonlinear wave equations: propagation of singularities, *Ann. Prob.* 16 (1988), 730–751.

[5] Cairoli, R. & Dalang, R.C., Optimal switching between two Brownian motions, Proc. AMS Summer Research Institute on Stochastic Analysis at Cornell University (1994, 10 pages, to appear).

[6] Csörgö, M. & Révész, P. (1981), On the non-differentiability of the Wiener sheet. *Contributions to Probability,* 143-150. Eds. Gani, J. and Rohatgi, V. Academic Press.

[7] Dalang, R. & Walsh, J.B., The Sharp Markov Property of the Brownian Sheet and Related Processes. *Acta Mathematica* 68, No.3-4, 1992, 153-218.

[8] Dalang, R. & Walsh, J.B., The sharp Markov property of Levy sheets. *Annals of Probab.* 20, 1992, 591-626.

[9] Dalang, R.C. & and Walsh, J.B., Geography of the Level Sets of the Brownian sheet, *Prob. Th. Rel. Fields* 96 (1993), 153-176.

[10] Dalang, R.C. & and Walsh, J.B., The Structure of a Brownian Bubble, *Prob. Th. Rel. Fields* 96 (1993), 475-501.

[11] Donati-Martin, C. & Nualart, D., Markov property for elliptic stochastic partial differential equations. *Stochastics and Stochastic Reports* 46, 1994, 107-115.

[12] Dvoretzky, A., Erdös, P. & Kakutani, S., Nonincrease everywhere of the Brownian motion process, Proc. 4th Berkeley Symp. Math. Stat. and Probab. 2 (1961), 103-116.

[13] El Karoui, N. & Karatzas, I., General Gittins index processes in discrete time, *Proc. Nat. Acad. Sci. (USA)* 90 (1993), 1232-1236.

[14] El Karoui, N. & Karatzas, I., Dynamic allocation problems in continuous time (1993, preprint).

[15] Léandre, R. & Russo, F., Small stochastic perturbation of a non-linear stochastic wave equation, in: *Progress in probability* 31, Korezlioglu, H. & Ustunel, A.S. (eds) (1992), 285-332.

[16] Mandelbaum, A., Continuous Multi-armed Bandits and Multi-parameter Processes, *Ann. Probab.* 15 (1987), 1527-1556.

[17] Mandelbaum, A., Shepp, L. & Vanderbei, R.J., Optimal switching between a pair of Brownian motions, *Ann. Probab.* 18-3 (1990), 1010-1033.

[18] Miller, R.N., Tropical data assimilation with simulated data: The impact of the tropical ocean and global atmosphere thermal array for the ocean, *Journal of Geophysical Research* 95 (1990), 11,461-11,482.

[19] Mueller, C., Long time existence for the wave equation with a noise term (1992, preprint).

[20] Nualart, D. & Pardoux, E., Markov Field Properties of Solutions of White Noise Driven Quasi-Linear Parabolic PDEs. *Stochastics and Stochastic Reports* 48, 1994, 17-44.

[21] Orsingher, E., Randomly forced vibrations of a string, *Ann. Inst. H. Poincarré* XVIII (1982), 367-394.

[22] Stein, E.M., Singular Integrals and Differentiability Properties of Functions, Princieton University Press (1970).

[23] Paley, R.E., Wiener, N. & Zygmund, A., Note on random functions, *Math. Z.* 37 (1933), 647-668.

[24] Wilcox, C.H., The Cauchy problem for the wave equation with distribution data: an elementary approach, *The Amer. Math. Monthly* 98 (1991), 401-410.